The 'New Math' reform and pedagogical flows in Hungarian and French mathematics education

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In this paper, I present my PhD research (in progress) focusing on the mathematical reform movements of Hungary and France in the 1960s and '70s and their origins: especially how much they were influenced by the international "New Math" movement, and by the mathematical and mathematics education traditions of these countries. I consider different aspects of the reform: curricula, textbooks, teaching practices. I look for their general characteristics, in the sense of the Pedagogical Flow approach (Schmidt et al., 1996), the leading principles behind these characteristics and the historical-cultural origins of these principles. In this paper, I present elements of this research, focusing on the example of the Pythagorean theorem.

Keywords: Pedagogical flow, New Math, Pythagorean theorem, textbook analysis, comparative research.

INRODUCTION

Although the reform introduced by Tamás Varga during the 1960s and '70s is generally recognized by the Hungarian mathematics education community as a key moment in the history of Hungarian mathematics education, having an important influence and keeping its values until today, its detailed historical or didactical analysis is lacking [1]. In my research, I consider Varga's work in its international context, comparing to the French "Mathématiques Modernes" reform of the same period: one of the most influential reforms during the international "New Math" movement, and also one of the well described ones, thanks to French historians of mathematics education (e.g., d'Enfert & Kahn, 2011). I attempt to describe general characteristics of these reforms, looking for coherent aspects between their different elements. Beyond common characteristics of the two countries' reforms, issued from the international discourse, I attempt to underline differences, and to show how they ensue from cultural traditions of the countries in question. I focus especially on mathematicians' conceptions about the nature of mathematics and its teaching.

I apply the term "pedagogical flow" in accordance with (Schmidt et al., 1996), where the international research group preparing the TIMSS studies introduces it to describe general characteristics of the mathematics educational system of a country which are present in different elements of mathematics education (as in curricula, textbooks, teaching practices). However, the cultural background of these characteristics, even if supposed, is hardly explored in (Schmidt et al., 1996); and the general model used by this research, the model SMSO which presents interrelations between the different levels of an educational system in detail, does not take into account the cultural, political, social or scientific background. Another model, the levels of codetermination of Chevallard (2002) takes these aspects into account, but integrates them into one hierarchical and linear system, and supposes that broader institutions like society or the scientific community of a country determine the lower levels of the system, such as the teaching of mathematics, for example.

In my research, I take into account complex interrelations between different elements (e.g., political, social, scientific and cultural) of the historical context, and the characteristics of the reform in each country. I focus more in detail on the epistemological background of mathematics and its teaching, expressed by a community of mathematicians in each studied country (communities which were particularly influential in the "New Math" period, as it is showed in the historical part of the study). By analysing written documents of the reforms: curricula, textbooks, teachers' handbooks and also the teaching practices suggested by these documents, I try to show how the conceptions expressed by mathematicians about mathematics and its teaching appear in the didactical characteristics of the reforms. In this paper, I focus on the example of the geometry curricula and, in particular, the Pythagorean theorem.

The "New Math" reform period is particularly suitable for this kind of research for several reasons. First, the "New Math" reforms are profound reforms, transforming curricula, resources and attended teaching practices in a coherent way, in accordance to some leading principles [2]. Secondly, exactly because of the profound changes, characteristics of the reform and the underlying principles are often explained in detail to inform the teachers and the society. Thirdly, the wide international discourses of the period make the comparison easier.

Finally, even though the "New Math" period is already history, it is not very far-away, and has its influence until today. Comparison of the present research to some more modern studies about "pedagogical flows" (e.g., Schmidt et al., 1996, for France; Andrews & Hatch, 2001, for Hungary) allows us to suppose a certain continuity. In this sense, the research on the "New Math" reform of Hungary and France can contribute to a better understanding of the historical, cultural and epistemological background of pedagogical flow in these countries.

METHODOLOGY

The research consists of three major parts: a historical, an epistemological and a didactical part. The first part of the research, concerning the history of mathematics education, is based on existing historical studies about France and about the international discourse of the "New Math" period. Concerning Hungary, general works on the history of pedagogy, original official sources, as well as written and oral memories of Varga's colleagues are used.

For the second, epistemological part, writings of mathematicians influencing the reforms are analysed: publications and lectures about mathematics education, mathematics popularisation books and correspondences. I look for characteristics of these mathematicians' conception about mathematics, and their main principles about its teaching.

The third, main part of the research is based on analytical tools provided by French theoretical frameworks of mathematics education research. After a general analysis of the content and of the structure of the curricula, three chapters are chosen from the first 8 grades in Hungary, and from the first 9 grades in France (primary and middle-school in each case). The analysis contains 1) an analysis of the place and role of the chosen chapter in the curriculum, based on the "ecological approach" of the Anthropological Theory of Didactics (Artaud, 1997); 2) a structural, rhetorical and linguistic analysis of the textbooks and teacher's handbooks; 3) an analysis of the teaching practices suggested by these resources, based on the Theory of Didactical Situations of Brousseau (1998).

THE HISTORICAL CONTEXT

During the 1960's and 70's, the international "New Math" reform movement, starting from the US and from some countries in Western Europe, influenced mathematics education in many countries of the world. France was one of the leading countries in this movement. International and French research studies underline the role of the technological competition of the Cold War, of mathematicians' efforts to integrate elements of modern mathematics, of the psychological discourses (first of all around Piaget), of the development of the educational systems and of society in the "New Math" reforms (e.g., d'Enfert & Kahn, 2011; Kilpatrick, 2012). Similar processes can be shown concerning Hungary.

The French reform called "Mathématiques Modernes" was introduced in 1969 for secondary, and in 1970 for primary education, following the work of a national committee led by the mathematician Lichnerowicz, but also vivid debates in teachers' associations and different, mostly short term experimentations. A modification of the reform took place in 1977.

In the same period in Hungary, a reform project was led by Tamás Varga, inspired by experiments of different countries but also by some Hungarian mathematicians and psychologists, and based on a long experimentation process since 1963 (Varga, 1975). This project was selected by a ministerial committee as basis of the reform of mathematics education, and the new official curriculum was introduced in 1978.

MATHEMATICIANS' DISCOURSES ON MATHEMATICS EDUCATION

In the "New Math" period, mathematicians participated actively in influencing mathematics education. In France, several mathematicians, often members of, or near to the Bourbaki group, expressed their opinions (e.g., Dieudonné, Choquet or Lichnerowicz, the leader of the committee preparing the reform). They emphasize the importance of modern, unified formal language, abstraction, structures and the axiomatic-deductive method in mathematics education. According to them, structures of modern mathematics correspond perfectly to the structures of human thinking; therefore they suggest that students should be introduced as quickly as possible to the use of this language and methodology (see, e.g., Piaget et al., 1955) [3].

In Hungary, mathematicians also took an active role in the reform movement of the period in question. In my present research, I focus on a group of first-rate Hungarian mathematicians who were interested in education since the 1940's and had important influence on the later reforms: first of all L. Kalmár, R. Péter, A. Rényi, L. Surányi, but also Hungarian thinkers living abroad: G. Pólya and I. Lakatos.

The analysis of their diverse writings (Gosztonyi, 2012) shows that these Hungarian mathematicians' image of mathematics is in deep contradiction with the one represented by the Bourbaki-school. They see mathematics as a constantly developing and changing creation of the human mind, and this development is guided by series of problems. According to them, the source of mathematics is intuition and experience; mathematical activity is basically dialogical and teaching mathematics is a joint activity of the students and of the teacher, where the teacher acts as an aid in the students' rediscovery of mathematics. Excessive formalism is discouraged; formal language being also seen as a result of a development. Mathematics is presented as a creative activity closely related to playing and to the arts.

DIDACTICAL ANALYSIS OF THE REFORMS

The content and the structure of the curricula

Concerning the curricula, both reforms aim to introduce new chapters from modern mathematics (such as set theory, logic, topology etc.), and to present mathematics as an integrated science (not "counting and measuring" as before the reform). But the way of realising this, the structure of the curricula is very different in the two countries; while the French curriculum is strictly hierarchic and linear, based on set theory, the Hungarian curriculum contains five big topics which are present in parallel during all the curriculum and interact with each other in a dialectic way: 1) sets and logic 2) arithmetic and algebra 3) relations, functions and series 4) geometry and measure and 5) combinatorics, probability and statistics.

In the followings I briefly present the case of the geometry curricula and that of the Pythagorean theorem.

In the French geometry curriculum of 1969 and 1970, an important break is marked between the lower grades (until 7th grade) and the last two years of the middle-school (8th and 9th grade). In the lower grades, geometry has minor importance, and is not recognised as 'veritable mathematics': the related chapters of the curriculum are named "observations of physical objects" and "practical exercises". The curriculum underlines that the study of 'veritable geometry' starts from the 8th grade, as an example of axiomatic thinking. Axioms and notions have to be introduced via physical observations, but once they are admitted, they have to be clearly distinguished from the physical word and every further theorem has to be deduced by formal demonstrations.

The study of geometry follows the axiomatic construction of real numbers, and is based on this last notion. Classical synthetic geometry is completely eliminated: the main aim of this geometry curriculum (in accordance with Bourbaki's construction of mathematics, where geometry is not an autonomous domain but part of topology) is not to study geometrical figures but to construct an algebraic tool to describe first the affine, then the Euclidian plane and space. Principal notions are projections, vectors, frames, transformations etc.

In this French curriculum, the Pythagorean Theorem is of limited importance: it is integrated in a bigger chapter about the Euclidian plane, as an algebraic consequence of a property of the orthogonal projection, and contributes to the construction of the notion of an orthogonal frame [4].

The Hungarian curriculum links geometry to other domains of modern mathematics (e.g., to set theory; to functions by transformations treated as movements; to combinatorics by discrete geometry), but not in a hierarchic, rather in a dialectic way. The visual nature of geometry plays an important role: geometry offers intuitive examples to treat problems of the other above mentioned domains. Although coordinate-systems are introduced, the studied geometry is mainly synthetic and concerns figures and their properties, transformations and symmetries.

The curriculum emphasises continuity; physical world experiences are present until the end of the middle-school, in a dynamic relation with argumentations and proofs on ideal figures. There is no completely axiomatic geometry in the Hungarian curriculum.

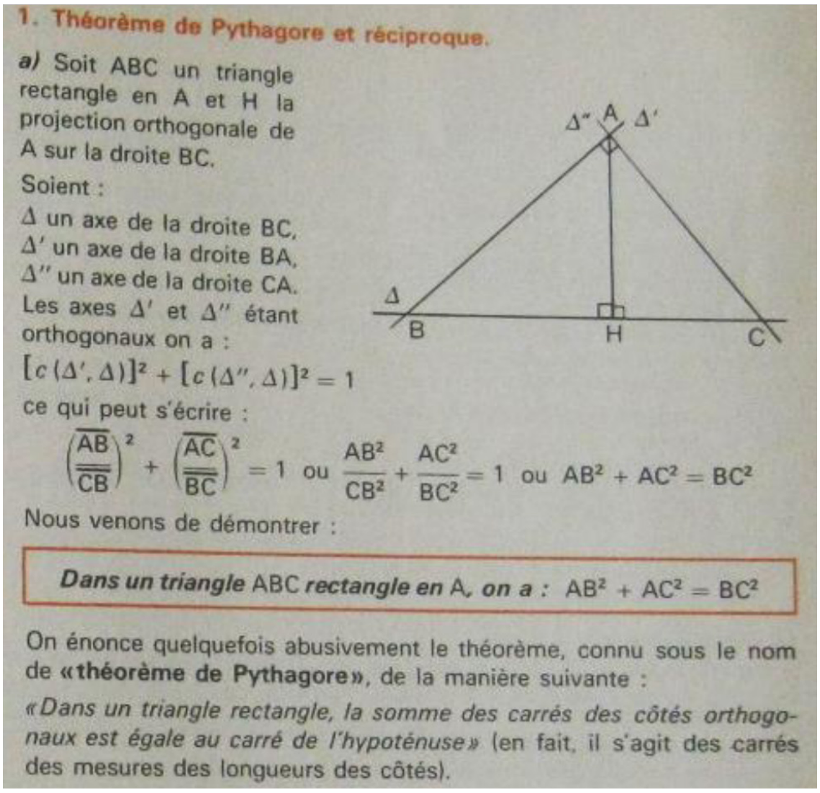
The Pythagorean theorem plays a significant role in this curriculum, not only as an important and useful property of right-angled triangles, but also as one of the first theorems, where students can discover the significance of proving.

The textbooks, teacher's handbooks and the attained teaching practices

In France, there is a great diversity of textbooks and of related handbooks, but some general tendencies can be observed. In Hungary in the period in question, there is only one obligatory series of textbooks. Here I present some structural, rhetorical and linguistic characteristics of middle-school textbooks, their suggestions about teaching practices; treating the example of the Pythagorean theorem in detail.

French middle-school textbooks, according to the curriculum, emphasise the initiation of students in the precise use of mathematical language. The first two years' books give also some natural language examples and describe some physical experiments; the second two years' books contain mainly formal mathematical discourse in an axiomatic-deductive form, followed by some "exercises" at the end of each chapter.

Figure 1 illustrates a typical treatment of the Pythagorean theorem in one of the French textbooks of the period. The demonstration is purely algebraic, requires developed formal and theoretical knowledge. The figure is only an illustration and what it represents is not really a triangle, rather three lines projected on each other. The textbooks, in accordance with the curricula, present the Pythagorean theorem as an element of a big theoretical system, constructing an algebraic tool to describe the plane and the space.



1. Théorème de Pythagore et réciproque.

a) Soit ABC un triangle rectangle en A et H la projection orthogonale de A sur la droite BC.

Soient :

Δ un axe de la droite BC,
Δ' un axe de la droite BA,
Δ'' un axe de la droite CA.

Les axes Δ' et Δ'' étant orthogonaux on a :

$[c(\Delta', \Delta)]^2 + [c(\Delta'', \Delta)]^2 = 1$

ce qui peut s'écrire :

$$\left(\frac{\overline{AB}}{\overline{CB}}\right)^2 + \left(\frac{\overline{AC}}{\overline{BC}}\right)^2 = 1 \text{ ou } \frac{AB^2}{CB^2} + \frac{AC^2}{BC^2} = 1 \text{ ou } AB^2 + AC^2 = BC^2$$

Nous venons de démontrer :

Dans un triangle** ABC **rectangle en** A, **on a : $AB^2 + AC^2 = BC^2$

On énonce quelquefois abusivement le théorème, connu sous le nom de **«théorème de Pythagore»**, de la manière suivante :

«Dans un triangle rectangle, la somme des carrés des côtés orthogonaux est égale au carré de l'hypoténuse» (en fait, il s'agit des carrés des mesures des longueurs des côtés).

Figure 1: (Fauverge, Jeanmot & Rieu, 1976, p. 163) Handbook for the 9th grade

This purely mathematic, deductive treatment corresponds to a lecture form in education: a direct transmission of institutional knowledge by teachers, and passive understanding by students (as we can understand from several textbooks' introduction). Although the introductions of the textbooks emphasise the importance of modern pedagogical methods and students' activity, they give only some general pedagogical indications and little help to concrete pedagogical practice. From the point of view of Brousseau's theory, these textbooks offer little *adidactical potentiality*: occasions to situations when students would engage in the construction of their own mathematical knowledge.[5]

Hungarian middle-school textbooks of the period are different from the presented French ones in several aspects. As we can see in the example presented below, the books contain a number of non-mathematical illustrations and didactical signs (the STOP sign means for example, as it explained in the introduction, that the reader should stop and think about the asked question). They also contain, in every grade, fictive dialogues of students to introduce new knowledge. The related teacher's handbook proposes to provoke similar discussions in the class. The dialogues are guided by a series of problems.

In the case of the Pythagorean theorem (see Figure 2), the first problem concerns the length of a rope stretched across the classroom, so that a student with a given height could stand under it. Students first estimate the result, and then solve the problem by experiment and measurements. The second problem is similar, but instead of the classroom, it concerns the bigger sports hall where students can't perform real experiments. The question is whether the difference between the length of the cord and that of the hall is bigger or smaller than in the case of the first problem. They first try to solve the problem by modelling and measuring, but the approximate result obtained this way isn't precise enough to answer the original question. Then they look for another method to solve the problem, "only with the help of calculations".

At that point, the handbook suggests finding a relation between the sides of a right-angled triangle, and introduces the figure of a classical geometrical proof of the Pythagorean theorem. Even the proof of the theorem is problematised, interrupted by questions and by discussions of students studying the figure. Finally, the theorem is applied to solve the original problem, as well as other problems.

The teaching practice suggested by the teacher's handbook and illustrated in the dialogues of textbooks is a kind of "guided discovery" process: students are guided through a series of problems, while continuing a dialogue between each other and with the teacher about the problems. Intuition, visuality and experiences play important role in this discovery process.

From the point of view of the theory of Brousseau, it is difficult to determine whether this work of students can be called *adidactic*: they rarely work autonomously, without the teacher's intervention (which is a necessary condition of the classical notion of an adidactical situation); nevertheless they take important responsibility in the process of constructing mathematical knowledge. So, the "guided discovery" teaching practice can be interpreted as involving an adidactical character of student's work, even if it doesn't correspond exactly to the classical notion of adidacticity.

CONCLUSION

The different presented aspects of mathematics education: the content and the structure of the curricula, the form of textbooks, and the attended teaching practices show great coherence both in the case of Hungary and of France. This observed coherence allows us to talk about "pedagogical flows" in the sense of (Schmidt et al., 1996).

Some common characteristics of the two reforms can be observed which may take their origin from the international discourses of the New Math period (like the ambition to present mathematics as a new, coherent subject; the emphasis on 'mathematical thinking'; new topics introduced like set theory or logic; the use of manipulative tools, especially in primary school). But there are also some important differences between the two countries, and the analysis of the mathematicians' principles let us suppose that they can be traced back to some mathematical traditions of these countries.

In France, such characteristics are the focus on big theoretical systems and on the strict hierarchical structure of mathematics, the emphasis on the axiomatic-deductive method and on the formal language

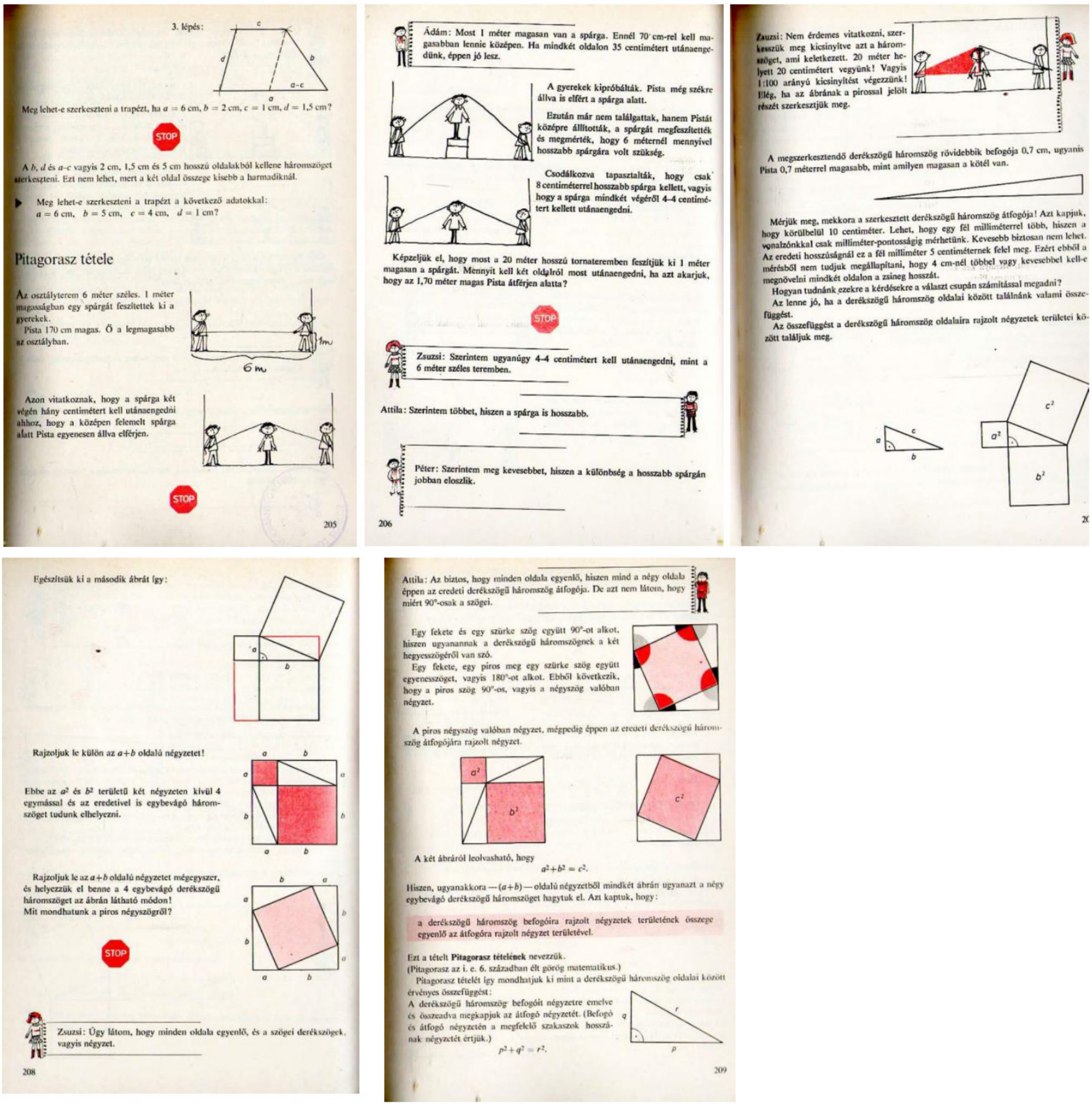


Figure 2: (Kovács, Sz. Földvári & Szeredi, 1980 pp. 205–209.) Handbook for the 7th grade

of mathematics. A principal aim of mathematics education is to initiate students into the knowledge and the methodology established by 'modern mathematics', considered as an ideal of human thinking. A tension can be observed between mathematical and pedagogical ambitions: although teacher's handbooks suggest to middle-school teachers to use some modern methods of active pedagogy, they offer little concrete suggestions to their realisation; the textbooks mostly correspond to the lecture form as a typical teaching practice.

In Hungary, the emphasis is more on the natural development of students' mathematical thinking and problem solving skills. The curriculum content is diversified and different topics interact dialectically, presenting the developmental, rather than the hierarchical nature of mathematics. Varga's curriculum is very careful with introducing formal language, and relies on empirical knowledge and on manipulative tools even on higher levels of mathematics education. A typical teaching form is the dialogue between the teacher and the students while they participate in a common discovery process based on series of problems.

To summarize, the "New Math" reforms of the two countries represent two, almost paradigmatic cases of mathematics education, related to different mathematical traditions and different epistemologies: a "bourbakist" view in the French case and a "heuristic" or "lakatosian" view in the Hungarian case.

The "New Math" is already history, and this research couldn't even attempt to have access to practices in ordinary classes. Curricula and resources changed in important ways in both of the countries since the 1970s, following social debates and didactical research among other factors (actually, the changes seems to be even more important in the French than in the Hungarian case). But, as I mentioned above, a comparison with results of other research works based on current classroom observations, confirms that several observed characteristics remain present both in French and in Hungarian mathematics education (see, e.g., Schmidt et al., 1996; Andrews & Hatch, 2001). The analysis of the "New Math" reforms may complete these existing observations, provide basis for further ones, and contribute to understanding the complex interrelations within a country's pedagogical flow more profoundly.

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ENDNOTES

1. Short commemorations with his colleagues are accessible, e.g., in the proceedings of the yearly organised 'Varga Tamás Days' (http://mathdid.elte.hu/html/vtn.html). In English see for example, Szendrei (2007).

2. This research takes into account the *intended* and *potentially implemented* curricula, described in the official documents and in the textbooks and teacher's handbooks. I don't consider the *implemented* curricula, the practice of ordinary teachers in the period, which can be fundamentally different from the intended practices.

3. At the same time, the teachers' association, also influential in the debates around the reform, and convinced by the importance of reforming the content of the curricula, emphasises also the use of modern, active pedagogical methods.

4. The next French curriculum, of 1977, doesn't ask the complete axiomatic construction of real numbers or geometry in the middle-school any more, and emphasises the practice of proof rather than axiomatisation. The mathematical organisation of the geometry curriculum remains similar to the preceding one, however the curriculum provides broader liberty in the organisation of textbooks and in the practice of teachers.

5. This contradiction between pedagogical ambitions and their realisation can be interpreted in the con-

text of the debates mentioned in note 3. The observed tension between mathematical and pedagogical ambitions probably contributes to the emergence of French didactical researches during the 1970's. The textbooks related to the new reform of 1977 follow the development of the debates: although some of them remain similar to the preceding ones, new textbooks appear with more developed pedagogical suggestions, e.g. in problem solving, and with a more classical treatment of geometry, among other things.